Ascham School

Mathematics Form 6 - 2 Unit Trial Examination

1999

July 1999

Time allowed: 3 Hours
Plus 5 minutes reading time.

Instructions

1. Attempt ALL questions
2. All questions are of equal value.
3. All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
4. Standard integrals are printed on page 10.
5. Board approved calculators may be used.
6. Answer each question in a *separate* writing booklet.

Question 1.

(a) Find the exact value of $\dfrac{\frac{2}{5}+\frac{2}{3}}{1-\frac{4}{15}}$ [1]

(b) Using the table of standard integrals, find $\int \frac{1}{\sqrt{x^2-9}}dx$ [1]

(c) Expand and simplify $(2x - 3)(x^2 + 5x + 2)$ [2]

(d) Find the exact value of $t^4 - t^2 + 1$ when $t = 2\sqrt{3}$ [2]

(e) Factorise $x^2 - y^2 + 2y - 2x$ [2]

(f) Solve for x $\dfrac{3x-1}{5x+1} = \dfrac{3x-2}{5x+2}$ [2]

(g) The roots of the equation $x^2 - 4x + 7 = 0$ are α and β. Evaluate $\alpha\beta^2 + \alpha^2\beta$ [2]

Question 2. Use a separate writing booklet.

(a) If $f(x) = \dfrac{1}{1+x^3}$

(i) Evaluate $f(1)$ and $f(2)$ [1]

(ii) Using the trapezoidal rule with three function values, find the approximate area under the curve between $x=0$ and $x=2$ [2]

(b) Evaluate $\log_4\left(\dfrac{1}{\sqrt{2}}\right)$ [2]

(c) Solve

(i) $x^2 + 6x - 16 = 0$ [2]

(ii) $x^2 + 6x - 16 \geq 0$ [1]

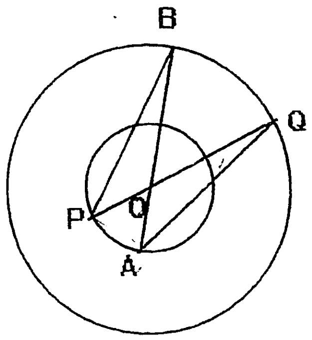
(d) Consider the parabola with equation $y = x^2 + 2x - 5$

(i) Express the equation in the form $(x - h)^2 = 4a(y - k)$ where a, h and k are constants. [2]

(ii) Write down the co-ordinates of the vertex and the equation of the directrix. [2]

Question 3. Use a separate writing booklet.

(a)



In the diagram, O is the centre of each of the circles. AOB and POQ are straight lines.

(i) Prove $\Delta OAQ \equiv \Delta OPB$

(ii) Hence prove that AQ = PB [4]

(b) Differentiate with respect to x

(i) $x^{0.2} - x + \frac{1}{3}$

(ii) $(5-x)^{-2}$

(iii) $\ln\frac{x}{2}$ [4]

(c) Find:

(i) $\int (2x+5)^6 dx$

(ii) $\int_{0.2}^{2} 3e^{5t} dt$, giving your answer correct to 3 significant figures [4]

Question 4. Use a separate writing booklet.

(a) Find the exact value of $\int_{\sqrt{e}}^{e^3} \frac{dx}{x}$ [2]

(b) For what values of k is the expression $x^2 - (1+k)x - (1-2k)$ positive definite ? [2]

(c) The quadrilateral PQRS where PQ//RS has coordinates P(-1, 0), Q(3, 2), R(4, 5) and S(0, 3)

(i) In your writing booklet draw a diagram showing this information.

(ii) Find the length of SR

(iii) Show that PQRS is a parallelogram

(iv) Find the equation of SR

(v) Hence show that the perpendicular distance from Q to SR is $\sqrt{5}$ units.

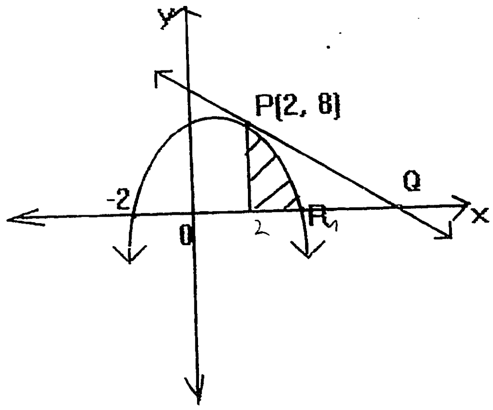
(vi) Find the area of PQRS.

[8]

Please turn over to question 5

Question 5. Use a separate writing booklet.

(a) The equation of the parabola shown is $y = 8 + 2x - x^2$. A tangent is drawn at the point P(2, 8).



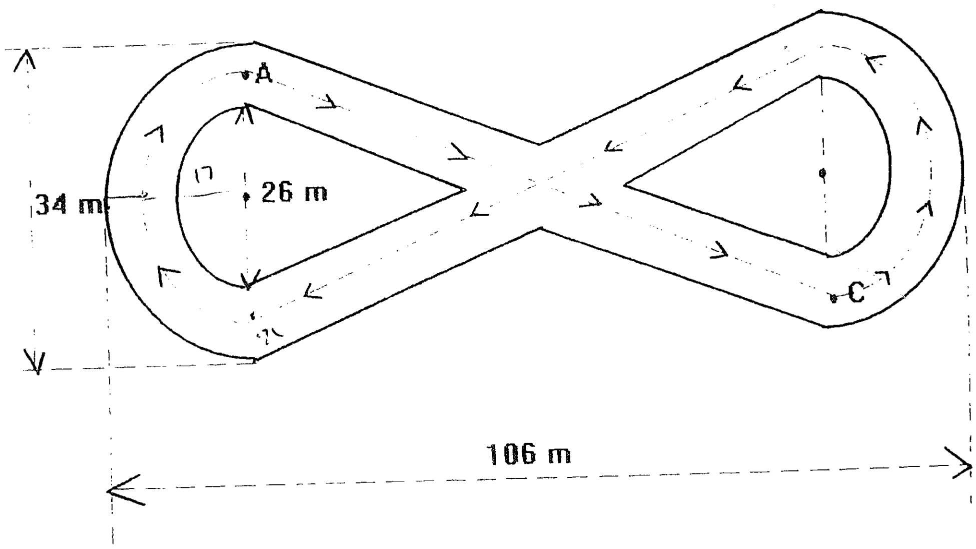
(i) Show that the equation of the tangent at P is $2x + y - 12 = 0$

(ii) State the co-ordinates of R and Q.

(iii) Find the size of the shaded area.

(iv) Hence determine the size of the area bounded by PQR **[8]**

(b) A speed skater does one lap of the track as shown, staying in the middle of the track all the way, including the semi-circular ends.



(i) Show that the length of AC is 78m

(ii) Show that the length of one lap around the track by the skater is given by $(30\pi + 156)$ metres. **[4]**

Question 6. Use a separate writing booklet.

(a) Find the value of $\cot\frac{\pi}{2}$ [1]

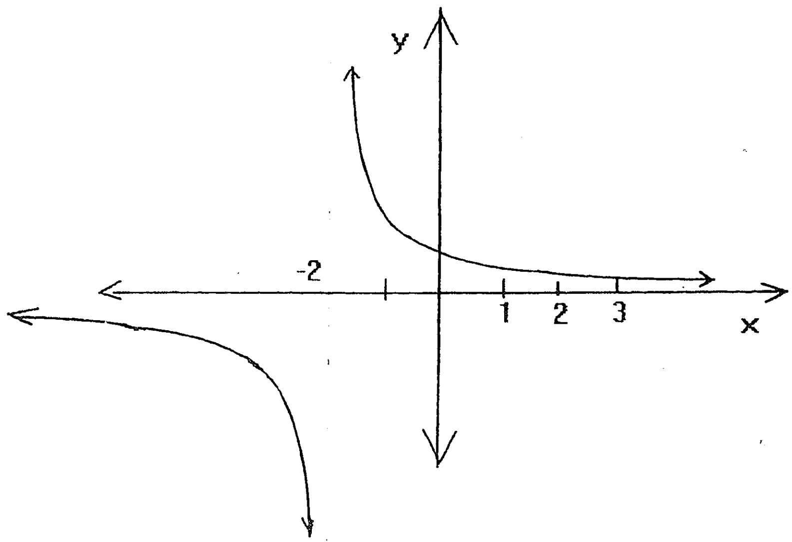
(b) Consider the curve y = (cosπx) -1

(i) State the period and range

(ii) Sketch the curve y = (cosπx) -1 for $0 \le x \le 2$

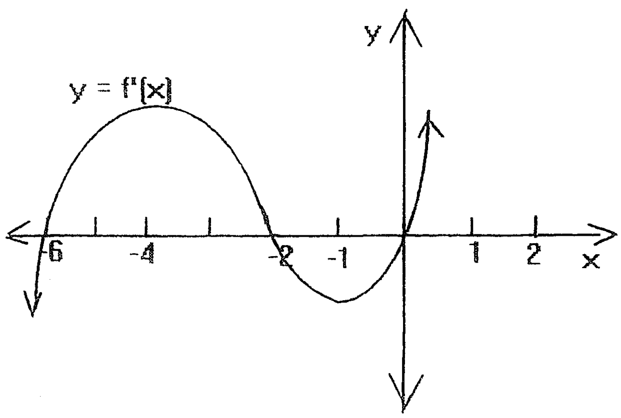
(iii) State the number of solutions to the equation cos πx = $\frac{1}{2}$ for $0 \le x \le 2$ [5]

(c) The hyperbola $y = \frac{1}{x+2}$ is sketched below. The area under the curve for $0 \le x \le 3$ is rotated about the x axis. Find the volume generated.



[3]

(d) The gradient function y = f '(x) of a curve y = f(x) is shown below.



(i) Describe the shape of the curve y = f(x) at x = -6, at x = -2 and at x = 0.

(ii) For what values of x is the curve y = f(x) increasing ?

(iii) In your writing booklet, sketch a possible curve for y = f(x)

[3]

Question 7. Use a separate writing booklet.

(a) Show that $\dfrac{4}{2+\sqrt{5}}-\dfrac{1}{9-4\sqrt{5}}$ is rational. **[2]**

(b) Consider the curve $y=(1+2x)e^{-2x}$

(i) Find the y-intercept.

(ii) Find where the curve crosses the x-axis.

(iii) Find any stationary points and determine their nature.

(iv) Show that there is a point of inflexion when $x=\dfrac{1}{2}$

(v) Discuss the behaviour of the curve as it approaches $\pm\infty$

(vi) Sketch the curve showing the above features.

[10]

Question 8.Use a separate writing booklet.

(a) The first and last terms of an arithmetic series are 5 and 165 respectively, and the sum of these terms is 1785. Find,

(i) the number of terms in the series

(ii) the common difference. **[4]**

(b) A retired woman decides to live off her savings. She has $70 000 and invests it at an interest rate of 6% p.a., compounded monthly. At the end of each month after interest has been received, she withdraws $M. The amount left at the end of the *nth* month after she has made her withdrawal is $$A_n$.

(i) Show that $A_1 = 70\,000\,(1.005) - M$ and that
$A_2 = 70\,000\,(1.005)^2 - M(1.005 + 1)$

(ii) Find an expression for the amount left after n months

(iii) Show that $M = \dfrac{70000(1.005)^n - A_n}{\left[\dfrac{1.005^n - 1}{0.005}\right]}$

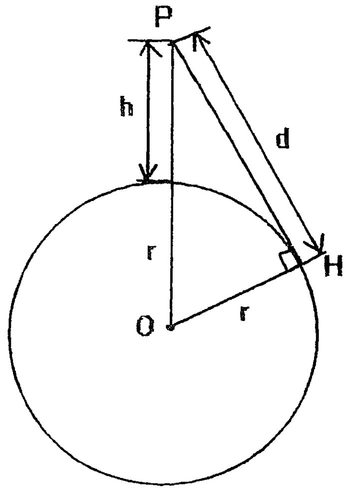
(iv) Find the monthly withdrawal if the woman has no money left after 10 years.

[8]

Question 9. Use a separate writing booklet.

(a) Find in exact form the area of a sector of a circle if it forms an angle of 20° at the centre and the radius of the circle is 5 cm.

(b) The diagram (which is not drawn to scale) represents the earth (assumed to be a sphere) and a balloonist at P, h metres above the surface. PH is the distance d, in metres, to the horizon, and r is the radius of the earth, also in metres.



(i) Show that $h = r\sqrt{1+\frac{d^2}{r^2}} - r$

(ii) When k is very small, say $k < 10^{-4}$, then $\sqrt{1+k}$ can be approximated by $1 + \frac{1}{2}k$.

Show that in such a case, $h = \frac{d^2}{2r}$ approximately.

(iii) If the balloonist at P has her horizon 40 km distant and the radius of the earth is 6 400 km,

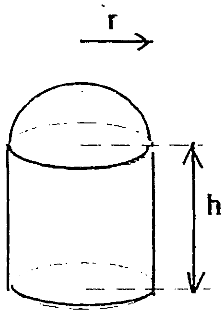
1. Show that the formula in (ii), $h = \frac{d^2}{2r}$, is valid to calculate h.

2. How many metres is the balloonist above the surface of the earth ?

[10]

Question 10. Use a separate writing booklet.

(a)



A nuclear reactor is to housed in a building consisting of a hemisphere surmounting a cylinder. This housing is to be constructed on an already existing concrete slab. It's volume is to be 576π cubic metres. The builder charges \$k / m^2 to build the cylindrical section and \$2k / m^2 to build the hemisphere.

(i) Write down an expression for the volume of the housing.
(Volume of a sphere = $\frac{4}{3}\pi r^3$, Volume of a cylinder = $\pi r^2 h$)

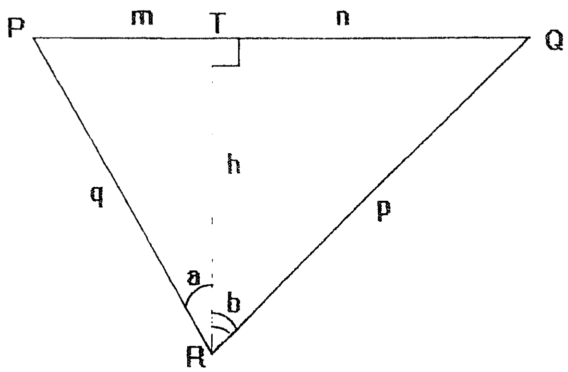
(ii) Show that the cost of construction \$C is given by $C = 2k\pi r(2r + h)$
(Surface area of a sphere = $4\pi r^2$, Curved surface area of a cylinder = $2\pi rh$)

(iii) Hence show that $C = \frac{8k\pi r^2}{3} + \frac{1152k\pi}{r}$

(iv) Hence find the minimum cost of the construction.

[8]

(b) Consider the diagram below.



(i) Use ΔPTR and ΔQTR to show that $\sin a \cos b + \sin b \cos a = \frac{(m+n)h}{pq}$

(ii) Show that $\sin(a+b) = \frac{(m+n)\sin(\frac{\pi}{2} - b)}{q}$

(iii) Hence prove that $\sin(a+b) = \sin a \cos b + \sin b \cos a$

[4]

End of test.

Question 1

a) $\dfrac{\frac{2}{5}+\frac{2}{3}}{1-\frac{4}{15}} = \dfrac{\frac{6+10}{15}}{\frac{11}{15}}$ ½

$= \dfrac{16}{15} \times \dfrac{15}{11}$

$= \dfrac{16}{11}$ or

$= 1\frac{5}{11}$ ½

b) $\displaystyle\int \frac{1}{\sqrt{x^2-9}}\,dx = \ln\left(x+\sqrt{x^2-9}\right)+C$ for $x > 3$ ✓

c) $(2x-3)(x^2+5x+2) = 2x^3+10x^2+4x-3x^2-15x-6$

$= 2x^3+7x^2-11x-6$

d) $t^4-t^2+1 = (2\sqrt{3})^4-(2\sqrt{3})^2+1$

$= 144-12+1$ ✓

$= 133$ ✓

e) $x^2-y^2+2y-2x = (x-y)(x+y)+2(y-x)$ ½

$= (x-y)(x+y)-2(x-y)$ ½

$= (x-y)(x+y-2)$ ✓

f) $\dfrac{3x-1}{5x+1} = \dfrac{3x-2}{5x+2}$

$(3x-1)(5x+2) = (3x-2)(5x+1)$ ½

$15x^2+6x-5x-2 = 15x^2+3x-10x-2$ ½

$x-2 = -7x-2$ ½

$8x = 0$

$x = 0$ ½

g) $\alpha\beta^2+\alpha^2\beta = \alpha\beta(\beta+\alpha)$ ½

$= 7\times 4$ ✓

$= 28$ ½

/2

Question 2.

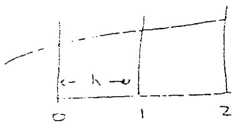
a) i) $f(1) = \dfrac{1}{1+1^3}$ $\quad f(2) = \dfrac{1}{1+2^3}$

$= \dfrac{1}{2}$ ½ $\quad = \dfrac{1}{9}$ ½

/1

ii) Area $= \dfrac{h}{2}\left(y_0+y_n+2y_1\right)$ ½

$= \dfrac{1}{2}\left(1+\dfrac{1}{9}+2\times\dfrac{1}{2}\right)$ ✓



Note $f(0) = \frac{1}{1}$
$= 1$

$= \dfrac{1}{2}\left(2\frac{1}{9}\right)$

$= 1\frac{1}{18}$ square units ½

/2

b) $\log_4\left(\dfrac{1}{\sqrt{2}}\right) = \log_4 2^{-\frac{1}{2}}$ ½

$= -\dfrac{1}{2}\,\dfrac{\log_2 2}{\log_2 4}$ ✓

$= \dfrac{-\frac{1}{2}}{2\log_2 2}$

$= \dfrac{-\frac{1}{2}}{2}$

$= -\frac{1}{4}$ ½

/2

c) i) $x^2+6x-16 = 0$

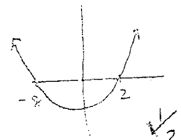
$(x+8)(x-2) = 0$ ✓

$\therefore x = -8$ or 2 ✓

ii) $x^2+6x-16 \geq 0$

$x \geq 2$ or $x \leq -8$

½



½

Question 2

d) i)

$y = x^2 + 2x - 5$

$y + 5 + 1 = x^2 + 2x + 1$ ✓

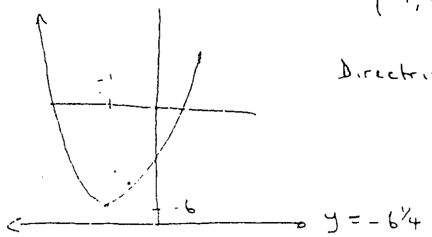
$y + 6 = (x+1)^2$ ✓

ie $(x+1)^2 = (y+6)$

ii) ie $h = -1$ $a = 1/4$ and $k = -6$

Vertex $(-1, -6)$ ✓

Directrix $y = -6\frac{1}{4}$ ✓



Question 3

a) i) In Δs OAQ and OPB

OP = OA (radii of the same circle) ½

OB = OQ (") ½

$\angle$POB = $\angle$AOQ (vertically opp. $\angle$'s) ✓

$\therefore$ ΔOPB $\equiv$ ΔOAQ (SAS) ✓

ii) $\therefore$ AQ = PB (corr. sides of $\equiv$ Δ's) ✓

b) i) $\frac{d}{dx}\; x^{0.2} - x + \frac{1}{3} = 0.2x^{-0.8} - 1$ or

$= \frac{1}{5}x^{-4/5} - 1$ or

$= \frac{1}{5\sqrt[5]{x^4}} - 1$ ✓

ii) $\frac{d}{dx}(5 - x)^{-2} = -2(5-x)^{-3} \times -1$ ½

$= 2(5-x)^{-3}$ or

$= \frac{2}{(5-x)^3}$ ✓

Question 3

b) iii) $\frac{d}{dx}\ln\left(\frac{x}{2}\right) = \frac{2}{x} \times \frac{1}{2}$ ✓

$= \frac{1}{x}$ ½ $1\frac{1}{2}$

c) i) $\int (2x+5)^6\, dx = \frac{(2x+5)^7}{7.2} + c$ ✓

$= \frac{(2x+5)^7}{14} + C$ ½ $1\frac{1}{2}$

ii) $\int_{0.2}^{2} 3e^{5t}\, dt = \left[\frac{3e^{5t}}{5}\right]_{0.2}^{2}$ ✓

$= \frac{3e^{10}}{5} - \frac{3e}{5}$

$= \frac{3}{5}(e^{10} - e)$ ½

$= 13214.24851...$ ½

$= 13200$ to 3 sig. fig ½ $2\frac{1}{2}$

Question 4

a) $\int_{\sqrt{e}}^{e^3} \frac{dx}{x} = \left[\ln x\right]_{\sqrt{e}}^{e^3}$ ½

$= \ln e^3 - \ln e^{1/2}$ ½

$= 3\ln e - \frac{1}{2}\ln e$ ½

$= 2\frac{1}{2}$ ½   2

b) A parabola is positive def^n if concave up and $\Delta < 0$

$a = 1 > 0$ $\therefore$ Concave up ½

and $\Delta = b^2 - 4ac$

$= [-(1+k)]^2 - 4 \times 1 \times [-(1-2k)]$ ½

$= 1 + 2k + k^2 - 4(1 - 2k)$

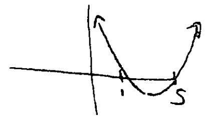
Question 4

b) con't.

$\Delta = 1 + 2k + k^2 + 4 - 8k$

$= k^2 - 6k + 5$

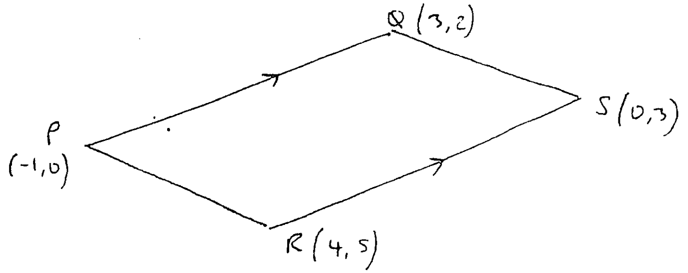
$= (k-5)(k-1)$ ✓½



$\therefore \Delta < 0$ for $1 < k < 5$

$\therefore$ curve is positive definite for $1 < k < 5$ ½

c) i)



$\overline{0}$

ii) $d_{SR} = \sqrt{(4-0)^2 + (5-3)^2}$ ½

$= \sqrt{16+4}$

$= \sqrt{20}$ ½ $\overline{1}$

iii) $RS \parallel PQ$ (given)

Furthermore $d_{PQ} = \sqrt{(-1-3)^2 + (0-2)^2}$ ½

$= \sqrt{(-4)^2 + (-2)^2}$

$= \sqrt{16+4}$

$= \sqrt{20}$ ½

$= d_{SR}$ (from ii) above)

$\therefore$ PQSR is a parm. (one pair of opp. sides is equal and parallel.) ✓ $\overline{2}$

5 Question 4

c iv) $M_{SR} = \frac{5-3}{4-0}$ ½

$= \frac{2}{4}$

$= \frac{1}{2}$ ½

$\therefore$ Equation of SR is

$y - 3 = \frac{1}{2}(x - 0)$ ½

$2y - 6 = x$

$0 = x - 2y + 6$ (or $y = \frac{1}{2}x + 3$) ½

v) Perpendicular Distance $= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$

$= \left|\frac{3-4+6}{\sqrt{1+4}}\right|$ ½

$= \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ ½

$= \sqrt{5}$ units ½ $\overline{1\frac{1}{2}}$

vi) Area of parm $= bh$

$= \sqrt{20}\sqrt{5}$ ½

$= \sqrt{100}$ ½

$= 10$ sq units ½ $\overline{1\frac{1}{2}}$

Question 5

a) i) $y = 8 + 2x - x^2$

$y' = 2 - 2x$ ½

$= 2 - 4$ at P

$= -2$ ½

$\therefore$ Equation of Tangent is

$y - 8 = -2(x - 2)$ ½

$y - 8 = -2x + 4$ ½

$2x + y - 12 = 0$ $\overline{2}$

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Question 5

a) ii) Q is pt. on tangent where $y = 0$

$\therefore 2x + 0 - 12 = 0$ ½

$2x = 12$

$x = 6$

ie $Q(6, 0)$ ½

R is pt. on parabola where $y = 0$

$0 = 8 + 2x - x^2$ ½

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$ ½

$\therefore x = 4$ or -2 ½

$R(4, 0)$ ½ as $x = -2$ is the other intercept

2

iii) Shaded Area $= \int_2^4 8 + 2x - x^2 \, dx$ ½

$= \left[8x + x^2 - \frac{x^3}{3} \right]_2^4$ ½

$= \left(32 + 16 - \frac{64}{3}\right) - \left(16 + 4 - \frac{8}{3}\right)$ ½

$= 9\frac{1}{3}$ sq. units ½

2

iv) Area PQR = Area of Δ - Shaded area

$= \frac{1}{2} \times 4 \times 8 - 9\frac{1}{3}$ ½

$= 6\frac{2}{3}$ sq. units ½

1

b) i) Twice thickness of track $= 34 - 26$

$= 8$ ½

$\therefore$ thickness of track $= 4$ ½

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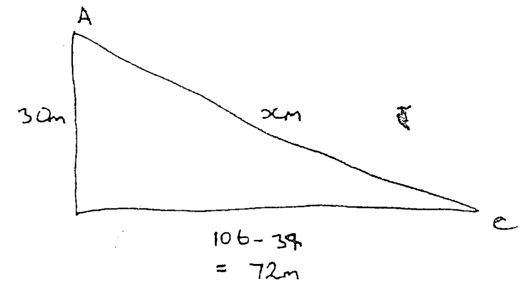


$x^2 = 30^2 + 72^2$

$= 6084$ ½

$\therefore x = 78$

$\therefore$ AC is 78m ½

3

ii) Length around track $= 2 \times 78 + 2\pi r$ ½

$= 156 + 2 \times \pi \times 15$ ½

$= 156 + 30\pi$ metres

1

Question 6

1a) $\cot \frac{\pi}{2} = \frac{\cos \pi/2}{\sin \pi/2}$ ½

$= \frac{0}{1}$

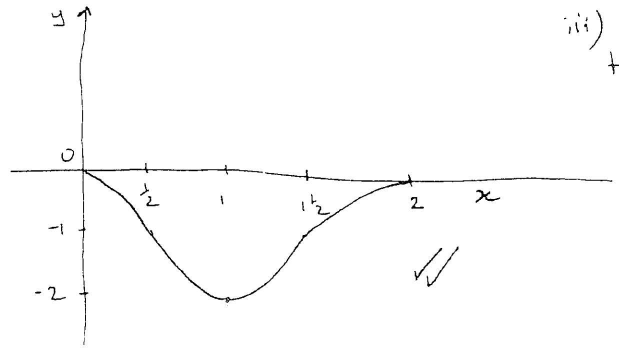
$= 0$ ½

1

b) i) Period $= \frac{2\pi}{\pi}$ Range $-2 \leq y \leq 0$

$= 2$

ii)



iii) There are 2 solutions to $\cos \pi x = \frac{1}{2}$ for $0 \leq x \leq 2$.

5

Question 6

i) Volume $= \pi \int_0^3 y^2\,dx$

$= \pi \int_0^3 \left(\frac{1}{x+2}\right)^2 dx$ ½

$= \pi \int_0^3 (x+2)^{-2}\,dx$ ½

$= \pi \left[\frac{(x+2)^{-1}}{-1.1}\right]_0^3$ ½

$= \pi\left[-5^{-1} - -2^{-1}\right]$ ½

$= \pi.\left[\frac{1}{2} - \frac{1}{5}\right]$ ½

$= \frac{5-2}{10}\pi$

$= \frac{3\pi}{10}$ cubic units. ½

3

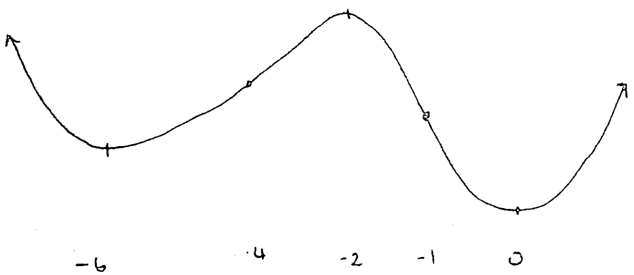
l) i) At $x=-6$ and $x=0$ $f(x)$ has a relative minimum ½

at $x=-2$ $f(x)$ has a relative maximum ½

ii) $y=f(x)$ is increasing for $f'(x)$ positive ie

$-6 < x < -2$ and $x > 0$ ✓

iii)



✓

3

possible $y=f(x)$

Question 7

a) $\frac{4}{2+\sqrt{5}} - \frac{1}{9-4\sqrt{5}} =$

$\frac{4}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} - \frac{1}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}} =$ ½

$\frac{8-4\sqrt{5}}{4-5} - \frac{9+4\sqrt{5}}{81-80} =$ ½

$\frac{8-4\sqrt{5}}{-1} - \frac{9+4\sqrt{5}}{1} =$

$-8+4\sqrt{5} - 9 - 4\sqrt{5} =$ ½

-17 which is rational ½

2

b) i) y intercept occurs when $x=0$ ie $y=(1+0)e^0$ ½

$=1$ ½

1

ii) crosses x axis when $y=0$ ie $1+2x=0$ since e^{-2x} never equals zero ½

$2x=-1$

$x=-\frac{1}{2}$ ½

ie curve crosses x axis at $(-\frac{1}{2}, 0)$

1

iii) $y = (1+2x)e^{-2x}$

$y' = (1+2x)e^{-2x} \times -2 + e^{-2x} \times 2$ ✓

$= -2e^{-2x}(1+2x) + 2e^{-2x}$

$= -4xe^{-2x}$ ½

For stationary points $0 = -4xe^{-2x}$ ½

$\therefore x = 0$ since $e^{-2x} \neq 0$ ½

$y'' = -4xe^{-2x} \times -2 + e^{-2x} \times -4$

$= 8xe^{-2x} - 4e^{-2x}$ ½

$= 4e^{-2x}(2x-1)$ ½

when $x=0$ $y''<0$ $\therefore$ (0,1) is a maximum ½

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Q7 b) iv) Possible points of inflexion occur when $y''=0$ ½

ie $4e^{-2x}(2x-1)=0$

ie $2x-1=0$ since $e^{-2x} \neq 0$ (ever!) ½

$x=\frac{1}{2}$

$f''(\frac{1}{2}-\varepsilon)<0$

$f''(\frac{1}{2})=0$

$f''(\frac{1}{2}+\varepsilon)>0$

½ ∴ $(\frac{1}{2}, 2e^{-1})$ is a pt. of inflexion ½

2

v) $y=(1+2x)e^{-2x}$

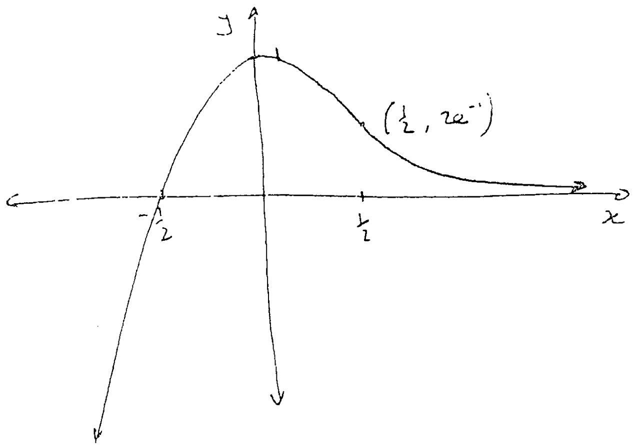
$=\frac{1+2x}{e^{2x}}$

As $x\to\infty$ $y\to 0^+$ ∵ e^{2x} gets v. big, quickly ½

As $x\to-\infty$ $y\to-\infty$ ∵ $e^{2x}>0$ and gets very big quickly ½

1

vi)



✓ 1

Q8.

a) i) $a=5$

$a+(n-1)d=165$

$S_n=1785$

$1785=\frac{n}{2}(a+l)$ ½

$=\frac{n}{2}(5+165)$ ½

$=\frac{n}{2}\times 170$

$=85n$ ½

$n=21$ ∴ There are 21 terms ½

2

ii) $165=5+20d$ ½

$160=20d$ ½

$d=8$ ∴ the common difference is 8. ✓

2

b) i) 6% p.a = 0.5% per month

$=0.005$ /month ✓

∴ $A_1=70000(1.005)-M$

and $A_2=[70000(1.005)-M](1.005)-M$ ✓

$=70000(1.005)^2-M(1.005)-M$ ✓

$=70000(1.005)^2-M(1.005+1)$

3

ii) $A_n=70000(1.005)^n-M(1.005^{n-1}+1.005^{n-2}+\dots+1)$ ✓

1

iii) $M(1+\dots+1.005^{n-2}+1.005^{n-1})=70000(1.005)^n-A_n$ ✓

$M\times\frac{1(1.005^n-1)}{0.005}=70000(1.005)^n-A_n$ ✓

∴ $M=\frac{70000(1.005)^n-A_n}{\left[\frac{1.005^n-1}{0.005}\right]}$

2

iv) $M=\frac{70000(1.005)^{120}-0}{\left[\frac{1.005^{120}-1}{.005}\right]}$ ✓ $=\$777.14$ to nearest cent. ✓

2

F6 2unit Trial – Ascham 1999 July

a) Area $= \frac{20}{360}\pi r^2$ ½

$= \frac{1}{18} \times \pi \times 25$ ✓

$= \frac{25\pi}{18}$ sq. units ½ — 2

b) i) $(h+r)^2 = r^2 + d^2$ (Pythagoras theorem) ½

$h + r = +\sqrt{r^2+d^2}$ ½

$h = \sqrt{r^2+d^2} - r$ ½

$= r\sqrt{1 + \frac{d^2}{r^2}} - r$ ✓ — 2½

ii) $h = r\sqrt{1+\frac{d^2}{r^2}} - r$

$\doteqdot r\left(1 + \frac{1}{2}\frac{d^2}{r^2}\right) - r$ ✓

$= r + \frac{d^2}{2r} - r$ ½

$= \frac{d^2}{2r}$ ½ — 2

iii) 1. The formula will be valid provided that $\frac{d^2}{r^2} < 10^{-4}$

$\frac{d^2}{r^2} = \frac{(40\,000)^2}{(6400\,000)^2}$ ✓

$= \frac{40000 \times 40000}{6400000 \times 6400000}$

$= \frac{16}{64 \times 6400}$

$= \frac{1}{4 \times 6400}$

$= 0.0000390625$ ✓

$< 0.0001 = 10^{-4}$ ½

∴ the formula is valid. ½

Question 9 b) iii) 2.

$h = \frac{d^2}{2r}$

$= \frac{(40000)^2}{2 \times 6400000}$ ✓

$= \frac{40000 \times 40000}{12800000}$

$= \frac{16000}{128}$

$= 125\text{ m}$ ✓

∴ the balloonist is 125 m above the earth. ½ — 2

Question 10

a) i) Volume of housing $= \pi r^2 h + \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \pi r^2 h + \frac{2}{3}\pi r^3$ ✓ — 1

ii) $S.A = \frac{1}{2} \times 4\pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\pi r h$ ½

Cost $= 2k \times 2\pi r^2 + k \times 2\pi r h$ ½ as cost of hemisphere is twice that of cylinder

$= 4k\pi r^2 + 2k\pi r h$ — 1

$= 2k\pi r(2r + h)$

iii) $576\pi = \pi r^2 h + \frac{2}{3}\pi r^3$ ½

$576 = r^2 h + \frac{2}{3} r^3$

$1728 = 3r^2 h + 2r^3$

$3r^2 h = 1728 - 2r^3$

$h = \frac{1728 - 2r^3}{3r^2}$ ½

∴ $C = 2k\pi r\left(2r + \frac{1728 - 2r^3}{3r^2}\right)$ ½

$= 2k\pi r\left(\frac{6r^3 + 1728 - 2r^3}{3r^2}\right)$ ½

15 Q10 a) iii)

$$C = 2k\pi r\left(\frac{1728+4r^3}{3r^2}\right)$$

$$= \frac{3456\pi r k}{3r^2} + \frac{8\pi r^4 k}{3r^2}$$

$$= \frac{1152\pi k}{r} + \frac{8k\pi r^2}{3}$$ ½

iv)

$$C' = -1152\pi k r^{-2} + \frac{8k\pi}{3}\cdot 2r$$ ½

$$= \frac{-1152\pi k}{r^2} + \frac{16\pi k r}{3}$$

$\therefore = 0$ when $\frac{1152\pi k}{r^2} = \frac{16\pi k r}{3}$ ½

$$3456\pi k = 16\pi k r^3$$

$$\frac{3456}{16} = r^3$$

$$r^3 = 216$$

$\therefore r = 6$ ½

$$C'' = 2304\pi k r^{-3} + \frac{16k\pi}{3}$$ ½

> 0 when $r = 6$ $\therefore$ minimum. ½

$\therefore$ minimum cost $= \frac{1152\pi k}{6} + \frac{8k\pi \cdot 36^{12}}{3}$ ½

$= 192k\pi + 96k\pi$

$= \$288k\pi$ ½

b) i)

$\sin a = \frac{m}{q}$ $\quad \cos a = \frac{h}{q}$

$\sin b = \frac{n}{p}$ $\quad \cos b = \frac{h}{p}$ ✓

$\therefore \sin a\cos b + \sin b\cos a = \frac{m}{q}\frac{h}{p} + \frac{n}{p}\times\frac{h}{q}$ ½

$$= \frac{(m+n)h}{qp}$$

16

F6 2 Unit Trial – Ascham 1999 July

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Q10 b) ii)

$$\frac{\sin(a+b)}{m+n} = \frac{\sin Q}{q} \quad \text{(Sine Rule)}$$ ½

$$= \frac{\sin(\frac{\pi}{2}-b)}{q}$$ ½

$$\therefore \sin(a+b) = \frac{(m+n)\sin(\frac{\pi}{2}-b)}{q}$$ 1

iii)

$$\sin\left(\frac{\pi}{2}-b\right) = \frac{h}{p}$$ ½

$$\therefore \sin(a+b) = \frac{(m+n)\frac{h}{p}}{q}$$ ½

$$= \frac{(m+n)h}{pq}$$ ½ 1½

$$= \sin a\cos b + \sin b\cos a.$$